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Effect of dust particles on ferrofluid heated and soluted from below

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Abstract

This paper deals with the theoretical investigation of the effect of dust particles on the thermosolutal convection in ferrofluid subjected to a transverse uniform magnetic field. Using linearized stability theory and normal mode analysis, an exact solution is obtained for the case of two free boundaries. For the case of stationary convection, non-buoyancy magnetization and dust particles have a destabilizing effect, whereas stable solute gradient has a stabilizing effect on the onset of instability. The critical wave number and critical magnetic thermal Rayleigh number for the onset of instability are also determined numerically for sufficiently large values of buoyancy magnetic parameter M_1 and results are depicted graphically. It is observed that the critical magnetic thermal Rayleigh number is reduced because the heat capacity of clean fluid is supplemented by that of the dust particles. The principle of exchange of stabilities is found to hold good for the ferrofluid heated from below in the absence of dust particles and stable solute gradient. The oscillatory modes are introduced due to the presence of the dust particles and stable solute gradient, which were non-existent in their absence. The sufficient conditions for the non-existence of overstability are also obtained. The paper also reaffirms the qualitative findings of earlier investigations which are, in fact, limiting cases of the present study.

Keywords: Ferrofluid; Thermosolutal convection; Magnetic field; Dust particles; Magnetization

1. Introduction

Ferrofluids are obtained by suspending submicron sized particles of magnetite in a carrier such as kerosene, heptane or water. These fluids not found in nature, behave as a homogeneous medium and exhibit interesting phenomena. In the last millennium, the investigation on ferrofluids attracted researchers because of the increase of applications in areas such as instrumentation, lubrication, vacuum technology, vibration damping, metals recovery, acoustics; its commercial usage includes vacuum feed-throughs for semiconductor manufacturing and related uses, pressure seals for compressors and blowers, engineering, medicine, chemical reactor and high-speed silent printers, etc. During the last half century, research on magnetic liquids has been very productive in many fields. Major perspectives are connected with a massive shocks and oscillation damping (earthquake, airbags), but the contemporary application concerned mostly seals and cooling of loudspeakers. Strong efforts have been undertaken to synthesize stable suspensions of magnetic particles with different performances in magnetism, fluid mechanics or physical chemistry.

Experimental and theoretical physicists and engineers gave significant contributions to ferrohydrodynamics and its applications [1–7]. In recent years, increasing attention has been focused on the study of ferrofluids. In many lubrication situations it is required to place the lubricant at a desired position and then retain it there. Therefore, ferrofluids have been successfully employed as lubricants in various hydrodynamically lubricated bearing. This motivated several research workers to analyze ferrofluid lubrication for different bearing situation under various simplifying assumptions. Chandra et al. [8], Kumar et al. [9,10] and Sinha et al. [11]

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Nomenclature

Latin symbols

a	particle radius	m
b	subscript; basic state	
$\mathbf{B}$	magnetic induction	T
B	magnitude of $\mathbf{B}$	T
C	solute concentration	kg
C_0	constant average concentration at the bottom surface $z=-d/2$	kg
C_1	constant average concentration at the upper surface $z=+d/2$	kg
C_a	average concentration $C_a=(C_0+C_1)/2$	kg
$C_{V,H}$	specific heat at constant volume and magnetic field	$kJ\cdot m^{-3}\cdot K^{-1}$
C_{pt}	specific heat of dust particles	$kJ\cdot m^{-3}\cdot K^{-1}$
d	thickness of the ferrofluid layer	m
$D/Dt=(\partial/\partial t+\mathbf{q}\cdot\nabla)$	the convective derivative	s^{-1}
$(\partial/\partial t+\mathbf{q}_d\cdot\nabla)$	the convective derivative analogous to dust particles	s^{-1}
$\mathbf{g}$	acceleration due to gravity $\mathbf{g}=(0,0,-g)$	$m\cdot s^{-2}$
$\mathbf{H}$	magnetic field intensity	$Amp\cdot m^{-1}$
$\mathbf{H}^{ext}$	external magnetic field intensity	$Amp\cdot m^{-1}$
$\mathbf{H}'$	the perturbation in magnetic field intensity	$Amp\cdot m^{-1}$
H	magnitude of $\mathbf{H}$	$Amp\cdot m^{-1}$
H_0	uniform magnetic field intensity	$Amp\cdot m^{-1}$
$K=6\pi\mu a$	Stokes drag coefficient	$kg\cdot s^{-1}$
$\hat{\mathbf{k}}$	unit vector in the z-direction	
k_x	the wave number along the x-direction	m^{-1}
k_y	the wave number along the y-direction	m^{-1}
k	the resultant wave number $k=\sqrt{k_x^2+k_y^2}$	m^{-1}
K_1	thermal conductivity	$W\cdot m^{-1}\cdot K^{-1}$
K_1'	solute conductivity	$W\cdot m^{-1} kg^{-1}$
K_2	$=-(\partial M/\partial T)_{H_0,T_a}$ the pyromagnetic coefficient	$Amp\cdot m^{-1}\cdot K^{-1}$
K_3	$=(\partial M/\partial C)_{H_0,C_a}$ the salinity magnetic coefficient	$Amp\cdot m^{-1}\cdot kg^{-1}$
m	mass of the dust particle	kg
$\mathbf{M}$	magnetization	$Amp\cdot m^{-1}$
$\mathbf{M}'$	the perturbation in the magnetization	$Amp\cdot m^{-1}$
M	magnitude of $\mathbf{M}$	$Amp\cdot m^{-1}$
M_0	the magnetization when magnetic field is H_0, temperature T_a and concentration C_a	$Amp\cdot m^{-1}$
N	number density of the dust particles	$1\cdot m^{-3}$
N_0	$=$ const. uniform particle distribution	$1\cdot m^{-3}$
p	the fluid pressure	$N\cdot m^{-2}$
p'	the perturbation in fluid pressure	$N\cdot m^{-2}$
$\mathbf{q}$	velocity of the ferrofluid	$m\cdot s^{-1}$
$\mathbf{q}'=(u,v,w)$	the perturbation in velocity on basic quiescent state $(0,0,0)$	$m\cdot s^{-1}$
$\mathbf{q}_d$	velocity of the dust particles	$m\cdot s^{-1}$
$\mathbf{q}_1'=(\ell,r,s)$	the perturbation in velocity $(0,0,0)$	$m\cdot s^{-1}$
t	time	s
T	temperature	K
T_0	constant average temperature at the bottom surface $z=-d/2$	K
T_1	constant average temperature at the upper surface $z=+d/2$	K
T_a	average temperature $T_a=(T_0+T_1)/2$	K

Greek letters

α	coefficient of thermal expansion	K^{-1}
α'	an analogous solvent coefficient of expansion	K^{-1}
β	a uniform temperature gradient $\beta=\lvert dT/dz\rvert$	$K\cdot m^{-1}$
β'	a uniform concentration gradient $\beta'=\lvert dC/dz\rvert$	$kg\cdot m^{-1}$
ν	kinematic viscosity	$m^2\cdot s^{-1}$
μ	dynamic viscosity (constant)	$kg\cdot m^{-1}\cdot s^{-2}$
μ_0	magnetic permeability of free space	$H\cdot m^{-1}$
ρ	fluid density	$kg\cdot m^{-3}$
ρ_0	mean density of the clean fluid	$kg\cdot m^{-3}$
$\chi=(\partial M/\partial H)_{H_0,T_a}$	the magnetic susceptibility	
θ	the perturbation in temperature T	K
γ	the perturbation in concentration C	kg
ρ'	the perturbation in density ρ	$kg\cdot m^{-3}$
∇	del operator	m^{-1}
σ	the growth rate	s^{-1}
Φ'	the perturbed magnetic potential	Amp

presented mathematical analyses of ferrolubrication for various configurations using Shliomis model.

An authoritative introduction to the research on magnetic liquid has been discussed in detail in the celebrated monograph by Rosensweig [12]. This monograph reviews several applications of heat transfer through ferrofluids. Such phenomenon is enhanced convective cooling having a temperature dependent magnetic moment due to magnetization of the fluid. This magnetization, in general, is function of the magnetic field, temperature and density of the fluid. Any variation of these quantities can induce a change in body force distribution in the fluid. This leads to convection in ferrofluids in the presence of magnetic field gradient. This mechanism is known as ferroconvection, which is similar to Bénard convection (Chandrasekhar [13]). The convective instability of a ferrofluid for a fluid layer heated from below in the presence of uniform vertical magnetic field has been considered by Finlayson [14]. He explained the concept of thermo-mechanical interaction on ferrofluids. Thermo-convective stability of ferrofluids without considering buoyancy effects has been investigated by Lalas and Carmi [15], whereas Shliomis [16] analyzed the linearized relation for magnetized perturbed quantities at the limit of instability. Kumar and Chandra [17] considered the flow of a magnetic fluid through a channel in the presence of a transversely applied magnetic field and studied the flow behavior for various parameters. The stability of a static magnetic fluid under the action of an external pressure drop has been studied by Polevikov [18], whereas the thermal convection in a magnetic fluid has been considered by Zebib [19]. The thermal convection in a layer of magnetic fluid confined in a two-dimensional cylindrical geometry has been studied by Lange [20]. Schwab et al. [21] investigated experimentally the Finlayson's problem in the case of a strong magnetic field and detected the onset of convection by plotting the Nusselt number versus the Rayleigh number. Then, the critical Rayleigh number corresponds to a discontinuity in the slope. Later, Stiles and Kagan [22] examined the experimental problem reported by Schwab et al. [21] and generalized the Finlayson's model assuming that under a strong magnetic field, the rotational viscosity augments the shear viscosity.

The Bénard convection in ferrofluids has been considered by many authors [23–40]. The ferrofluid has been considered to be clean in all the above studies. In many situations, the fluid is often not pure but contains dust particles or impurities. Saffman [41] has considered the stability of laminar flow of a dusty gas. Scanlon and Segel [42] have considered the effects of suspended particles on the onset of Bénard convection, whereas Sharma et al. [43] have studied the effect of suspended particles on the onset of Bénard convection in hydromagnetics and found that the critical Rayleigh number is reduced because the heat capacity of clean fluid is supplemented by that of the dust particles. The separate effects of suspended particles, rotation and solute gradient on thermal instability of fluids through a porous medium have been discussed by Sharma and Sharma [44]. The suspended particles were thus found to destabilize the layer. Palaniswamy and Purushotham [45] have studied the stability of shear flow of stratified fluids with fine dust and found the effects of fine dust to increase the region of instability. On the other hand, the multiphase fluid systems are concerned with the motion of a liquid or gas containing immiscible inert identical particles. Of all multiphase fluid systems observed in nature, blood flow in arteries, flow in rocket tubes, dust in gas cooling systems to enhance the heat transfer processes, movement of inert solid particles in atmosphere, sand or other particles in sea or ocean beaches are the most common examples of multiphase fluid systems. Naturally, studies of these systems are mathematically interesting and physically useful for various good reasons. The effect of dust particles on non-magnetic fluids has been investigated by many authors [46–49]. The main result from all these studies is that dust particles have destabilizing effect on the system and specific heat of fluid is greater than the specific heat of particles is the sufficient condition for the non-existence of overstability.

In view of the above investigations and keeping in mind the importance of ferrofluids, it is attempted to discuss the effect of dust particles on thermosolutal convection in a ferrofluid subjected to a vertical magnetic field. In this analysis, neutral impurities alone are considered. The present study can serve as a theoretical support for an experimental investigation e.g. evaluating the influence of impurities in a ferrofluid on thermohaline convection phenomena. This problem, to the best of our knowledge, has not been investigated yet.

2. Mathematical formulation of the problem

We consider an infinite, horizontal layer of thickness d of an electrically non-conducting incompressible Boussinesq ferrofluid embedded in dust particles heated and soluted from below. A uniform magnetic field H_0 acts along the vertical direction which is taken as z-axis. The temperature T and solute concentration C at the bottom and top surfaces $z = \mp\frac{1}{2}d$ are T_0, T_1 and C_0, C_1, respectively (see Fig. 1). Both boundaries are taken to be free and perfect conductors of heat. The gravity field $\mathbf{g} = (0, 0, -g)$ and uniform vertical magnetic field intensity $\mathbf{H} = (0, 0, H_0)$ pervade the system.

The mathematical equations governing the motion of ferrofluid for the above model are as follows:

The continuity equation for an incompressible ferrofluid is

$$\nabla \cdot \mathbf{q} = 0 \tag{1}$$

The momentum equation is

$$\rho_0\left[\frac{\partial}{\partial t} + (\mathbf{q}\cdot\nabla)\right]\mathbf{q} = -\nabla p + \rho\mathbf{g} + KN(\mathbf{q}_\mathrm{d} - \mathbf{q}) + \nabla\cdot(\mathbf{HB}) + \mu\nabla^2\mathbf{q} \tag{2}$$

Assuming a uniform particle size, a spherical shape, and small relative velocities between the fluid and dust particles, the presence of dust particles adds an extra force term in the

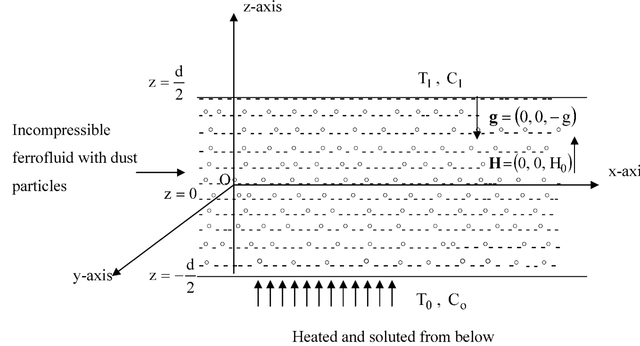

Fig. 1. Geometrical configuration.

equations of motion (2), proportional to the velocity difference between the dust particles and the fluid. Two additional simplifications are assumed in Eq. (2): we assume that the viscosity is isotropic and independent of the magnetic field. Both approximations simplify the analysis without changing the ultimate conclusion.

The density equation of state is taken as:

$$\rho = \rho_0\big[1 - \alpha(T - T_\mathrm{a}) + \alpha'(C - C_\mathrm{a})\big] \tag{3}$$

Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. Electrical forces are not considered. The buoyancy force on the particles is also neglected. This force is proportional to the quotient of ρ and the particle density, and an analysis for the case of free-free boundary conditions shows that its small stabilizing effect is neglected. Inter-particle reactions are also ignored since we assume that the distances between particles are quite large compared with their diameters. The effects due to pressure, gravity, on the particles are negligibly small, and therefore ignored. Under these restrictions, the equations of motion and continuity of the dust particles are

$$mN\left[\frac{\partial}{\partial t} + (\mathbf{q}_\mathrm{d}\cdot\nabla)\right]\mathbf{q}_\mathrm{d} = KN(\mathbf{q} - \mathbf{q}_\mathrm{d}) \tag{4}$$

$$\frac{\partial N}{\partial t} + \nabla\cdot(N\mathbf{q}_\mathrm{d}) = 0 \tag{5}$$

where mN is the mass of particles per unit volume.

The equations expressing the conservation of temperature and solute concentration in presence of dust particles are

$$\left[\rho_0 C_{\mathrm{V,H}} - \mu_0\mathbf{H}\cdot\left(\frac{\partial\mathbf{M}}{\partial T}\right)_{\mathrm{V},\mathbf{H}}\right]\frac{DT}{Dt} + \mu_0 T\left(\frac{\partial\mathbf{M}}{\partial T}\right)_{\mathrm{V},\mathbf{H}}\cdot\frac{D\mathbf{H}}{Dt} + mNC_{\mathrm{pt}}\left(\frac{\partial}{\partial t} + \mathbf{q}_\mathrm{d}\cdot\nabla\right)T = K_1\nabla^2 T \tag{6}$$

$$\left[\rho_0 C_{\mathrm{V,H}} - \mu_0\mathbf{H}\cdot\left(\frac{\partial\mathbf{M}}{\partial C}\right)_{\mathrm{V},\mathbf{H}}\right]\frac{DC}{Dt} + \mu_0 T\left(\frac{\partial\mathbf{M}}{\partial C}\right)_{\mathrm{V},\mathbf{H}}\cdot\frac{D\mathbf{H}}{Dt} + mNC_{\mathrm{pt}}\left(\frac{\partial}{\partial t} + \mathbf{q}_\mathrm{d}\cdot\nabla\right)C = K_1'\nabla^2 C \tag{7}$$

The partial derivatives of $\mathbf{M}$ are material properties which can be evaluated once the magnetic equation of state, such as Eq. (11) below, is known.

Maxwell's equations, simplified for a non-conducting fluid with no displacement currents, become

$$\nabla\cdot\mathbf{B} = 0, \qquad \nabla\times\mathbf{H} = \mathbf{0} \tag{8a,b}$$

In the Chu formulation of electrodynamics (Penfield and Haus [50]), the magnetic field, magnetization and the magnetic induction are related by

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \tag{9}$$

We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field, temperature and salinity, so that

$$\mathbf{M} = \frac{\mathbf{H}}{H}M(H, T, C) \tag{10}$$

The magnetic equation of state is linearized about the magnetic field, H_0, an average temperature, T_a, and the average salinity, C_a to become

$$M = M_0 + \chi(H - H_0) - K_2(T - T_\mathrm{a}) + K_3(C - C_\mathrm{a}) \tag{11}$$

where H_0 is the uniform magnetic field of the fluid layer when placed in an external magnetic field $\mathbf{H} = H_0^{\mathrm{ext}}\hat{\mathbf{k}}$ and $M_0 = M(H_0, T_\mathrm{a}, C_\mathrm{a})$.

The basic state is assumed to be quiescent state and is given by

$$\begin{gathered}
\mathbf{q} = \mathbf{q}_\mathrm{b} = \mathbf{0}, \quad \mathbf{q}_\mathrm{d} = (\mathbf{q}_\mathrm{d})_\mathrm{b} = \mathbf{0}, \quad \rho = \rho_\mathrm{b}(z), \quad p = p_\mathrm{b}(z) \\
T = T_\mathrm{b}(z) = -\beta z + T_\mathrm{a} \\
C = C_\mathrm{b}(z) = -\beta' z + C_\mathrm{a}, \quad \beta = \frac{T_1 - T_0}{d}, \quad \beta' = \frac{C_1 - C_0}{d} \\
\mathbf{H}_\mathrm{b} = \left[H_0 + \frac{K_2(T_\mathrm{b} - T_\mathrm{a})}{1+\chi} - \frac{K_3(C_\mathrm{b} - C_\mathrm{a})}{1+\chi}\right]\hat{\mathbf{k}} \\
\mathbf{M}_\mathrm{b} = \left[M_0 - \frac{K_2(T_\mathrm{b} - T_\mathrm{a})}{1+\chi} + \frac{K_3(C_\mathrm{b} - C_\mathrm{a})}{1+\chi}\right]\hat{\mathbf{k}} \\
H_0 + M_0 = H_0^{\mathrm{ext}}, \quad N = N_\mathrm{b} = N_0
\end{gathered} \tag{12}$$

Only the spatially varying parts of $\mathbf{H}_0$ and $\mathbf{M}_0$ contribute to the analysis, so that the direction of the external magnetic field is unimportant and the convection is the same whether the external magnetic field is parallel or antiparallel to the gravitational force.

3. The perturbation equations and normal mode analysis method

We shall analyze the stability of the basic state by introducing the following perturbations:

$$\begin{gathered}
\mathbf{q} = \mathbf{q}_\mathrm{b} + \mathbf{q}', \quad \mathbf{q}_\mathrm{d} = (\mathbf{q}_\mathrm{d})_\mathrm{b} + \mathbf{q}_1' \\
p = p_\mathrm{b}(z) + p', \quad \rho = \rho_\mathrm{b} + \rho' \\
T = T_\mathrm{b}(z) + \theta, \quad C = C_\mathrm{b}(z) + \gamma \\
\mathbf{H} = \mathbf{H}_\mathrm{b}(z) + \mathbf{H}' \quad \text{and} \quad \mathbf{M} = \mathbf{M}_\mathrm{b}(z) + \mathbf{M}'
\end{gathered} \tag{13}$$

These perturbations are assumed to be small and then the linearized perturbation equations become

$$L_1\rho_0\frac{\partial u}{\partial t} = L_1\left[-\frac{\partial p'}{\partial x} + \mu_0(M_0 + H_0)\frac{\partial H_1'}{\partial z} + \mu\nabla^2 u\right] - mN_0\frac{\partial u}{\partial t} \tag{14}$$

$$L_1\rho_0\frac{\partial v}{\partial t}=L_1\left[-\frac{\partial p'}{\partial y}+\mu_0(M_0+H_0)\frac{\partial H_2'}{\partial z}+\mu\nabla^2 v\right]-mN_0\frac{\partial v}{\partial t} \tag{15}$$

$$\begin{aligned}L_1\rho_0\frac{\partial w}{\partial t}=L_1\bigg[&-\frac{\partial p'}{\partial z}+\mu_0(M_0+H_0)\frac{\partial H_3'}{\partial z}+\mu\nabla^2 w\\&-\frac{\mu_0K_2\beta}{1+\chi}\{H_3'(1+\chi)-K_2\theta\}\\&+\frac{\mu_0K_3\beta'}{1+\chi}\{H_3'(1+\chi)+K_3\gamma\}\\&-\frac{\mu_0K_2K_3}{1+\chi}(\beta'\theta+\beta\gamma)+g\rho_0(\alpha\theta-\alpha'\gamma)\bigg]\\&-mN_0\frac{\partial w}{\partial t}\end{aligned} \tag{16}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{17}$$

$$\begin{aligned}&L_1\left[\{\rho C_1+mN_0C_{\mathrm{pt}}\}\frac{\partial\theta}{\partial t}-\mu_0T_0K_2\frac{\partial}{\partial t}\left(\frac{\partial\Phi_1'}{\partial z}\right)\right]\\&\quad=L_1\left[K_1\nabla^2\theta+\left\{\rho C_1\beta-\frac{\mu_0T_0K_2^2\beta}{(1+\chi)}\right\}w\right]\\&\qquad+mN_0\beta C_{\mathrm{pt}}w\end{aligned} \tag{18}$$

$$\begin{aligned}&L_1\left[\{\rho C_1'+mN_0C_{\mathrm{pt}}\}\frac{\partial\gamma}{\partial t}-\mu_0C_0K_3\frac{\partial}{\partial t}\left(\frac{\partial\Phi_2'}{\partial z}\right)\right]\\&\quad=L_1\left[K_1'\nabla^2\gamma+\left\{\rho C_1'\beta'-\frac{\mu_0C_0K_3^2\beta'}{(1+\chi)}\right\}w\right]\\&\qquad+mN_0\beta'C_{\mathrm{pt}}w\end{aligned} \tag{19}$$

where

$$\rho C_1=\rho_0C_{\mathrm{V,H}}+\mu_0K_2H_0,\quad \rho C_1'=\rho_0C_{\mathrm{V,H}}-\mu_0K_3H_0,$$
$$L_1=\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right) \tag{20}$$

Eqs. (10) and (11) yield

$$\left.\begin{aligned}&H_3'+M_3'=(1+\chi)H_3'-K_2\theta\\&H_3'+M_3'=(1+\chi)H_3'+K_3\gamma\\&H_i'+M_i'=\left(1+\frac{M_0}{H_0}\right)H_i'\quad(i=1,2)\end{aligned}\right\} \tag{21}$$

where we have assumed $K_2\beta d\ll(1+\chi)H_0$, $K_3\beta'd\ll(1+\chi)H_0$. Thus the analysis is restricted to physical situation in which the magnetization induced by temperature and concentration variations is small compared to that induced by the external magnetic field. Eq. (8b) means we can write $\mathbf{H}'=\nabla(\Phi_1'-\Phi_2')$, where Φ_1' is the perturbed magnetic potential and Φ_2' is the perturbed magnetic potential analogous to solute.

Eliminating u, v, p' between Eqs. (14)–(16), using Eq. (17), we obtain

$$\begin{aligned}&\left\{L_1\left(\rho_0\frac{\partial}{\partial t}-\mu\nabla^2\right)+mN_0\frac{\partial}{\partial t}\right\}\nabla^2 w\\&\quad=L_1\bigg[-\frac{\mu_0K_2\beta}{1+\chi}\nabla_1^2\left\{(1+\chi)\frac{\partial}{\partial z}(\Phi_1'-\Phi_2')-K_2\theta\right\}\\&\qquad+\frac{\mu_0K_3\beta'}{1+\chi}\nabla_1^2\left\{(1+\chi)\frac{\partial}{\partial z}(\Phi_1'-\Phi_2')+K_3\gamma\right\}\\&\qquad+\rho_0g\nabla_1^2(\alpha\theta-\alpha'\gamma)-\frac{\mu_0K_2K_3}{(1+\chi)}\nabla_1^2(\beta'\theta+\beta\gamma)\bigg]\end{aligned} \tag{22}$$

From Eqs. (21), we have

$$(1+\chi)\frac{\partial^2\Phi_1'}{\partial z^2}+\left(1+\frac{M_0}{H_0}\right)\nabla_1^2\Phi_1'-K_2\frac{\partial\theta}{\partial z}=0 \tag{23}$$

$$(1+\chi)\frac{\partial^2\Phi_2'}{\partial z^2}+\left(1+\frac{M_0}{H_0}\right)\nabla_1^2\Phi_2'-K_3\frac{\partial\gamma}{\partial z}=0 \tag{24}$$

Normal mode solution of all dynamical variables can be written as

$$\begin{aligned}&(w,\theta,\gamma,\Phi_1',\Phi_2')\\&\quad=\left[W(z,t),\Theta(z,t),\Gamma(z,t),\Phi_1(z,t),\Phi_2(z,t)\right]\\&\qquad\times\exp\mathrm{i}(k_xx+k_yy)\end{aligned} \tag{25}$$

Following normal mode analysis, the linearized perturbation dimensionless equations for thermosolutal convection in ferrofluid in the presence of dust particles become

$$\begin{aligned}&\left[L_1^*\left(\frac{\partial}{\partial t^*}-(D^2-a^2)\right)+f\frac{\partial}{\partial t^*}\right](D^2-a^2)W^*\\&\quad=aR^{1/2}L_1^*\left[(M_1-M_4)D\Phi_1^*-(1+M_1-M_4)T^*\right]\\&\qquad+aS^{1/2}L_1^*\left[(M_1'-M_4')D\Phi_2^*+(1-M_1'+M_4')C^*\right]\end{aligned} \tag{26}$$

$$\begin{aligned}&L_1^*P_r\left[(1+h)\frac{\partial T^*}{\partial t^*}-M_2\frac{\partial}{\partial t^*}(D\Phi_1^*)\right]\\&\quad=L_1^*(D^2-a^2)T^*+aR^{1/2}\left[L_1^*(1-M_2)+h\right]W^*\end{aligned} \tag{27}$$

$$\begin{aligned}&L_1^*P_s\left[(1+h')\frac{\partial C^*}{\partial t^*}-M_2'\frac{\partial}{\partial t^*}(D\Phi_2^*)\right]\\&\quad=L_1^*(D^2-a^2)C^*+aS^{1/2}\left[L_1^*(1-M_2')+h'\right]W^*\end{aligned} \tag{28}$$

$$D^2\Phi_1^*-a^2M_3\Phi_1^*-DT^*=0 \tag{29}$$

$$D^2\Phi_2^*-a^2M_3\Phi_2^*-DC^*=0 \tag{30}$$

where the following non-dimensional parameters are introduced:

$$t^*=\frac{\nu t}{d^2},\qquad W^*=\frac{Wd}{\nu}$$

$$\Phi_1^*=\frac{(1+\chi)K_1aR^{1/2}}{K_2\rho C_1\beta\nu d^2}\Phi_1,\qquad \Phi_2^*=\frac{(1+\chi)K_1'aS^{1/2}}{K_3\rho C_1'\beta'\nu d^2}\Phi_2$$

$$R=\frac{g\alpha\beta d^4\rho C_1}{\nu K_1},\qquad S=\frac{g\alpha'\beta'd^4\rho C_1'}{\nu K_1'}$$

$$T^*=\frac{K_1aR^{1/2}}{\rho C_1\beta\nu d}\Theta,\qquad C^*=\frac{K_1'aS^{1/2}}{\rho C_1'\beta'\nu d}\Gamma,\qquad a=kd$$

$$z^* = \frac{z}{d}, \qquad D = \frac{\partial}{\partial z^*}$$
$$P_r = \frac{\nu}{K_1}\rho C_1, \qquad P_s = \frac{\nu}{K_1'}\rho C_1'$$
$$M_1 = \frac{\mu_0 K_2^2 \beta}{(1+\chi)\alpha\rho_0 g}, \qquad M_1' = \frac{\mu_0 K_3^2 \beta'}{(1+\chi)\alpha'\rho_0 g}$$
$$M_2 = \frac{\mu_0 T_0 K_2^2}{(1+\chi)\rho C_1}, \qquad M_2' = \frac{\mu_0 C_0 K_3^2}{(1+\chi)\rho C_1'}$$
$$M_3 = \frac{1+M_0/H_0}{1+\chi}, \qquad M_4 = \frac{\mu_0 K_2 K_3 \beta'}{(1+\chi)\alpha\rho_0 g}$$
$$M_4' = \frac{\mu_0 K_2 K_3 \beta}{(1+\chi)\alpha'\rho_0 g}, \qquad M_5 = \frac{M_4}{M_1} = \frac{M_1'}{M_4'} = \frac{K_3\beta'}{K_2\beta}$$
$$\tau = \frac{m\nu}{Kd^2}, \qquad L_1^* = \left(\tau\frac{\partial}{\partial t^*}+1\right), \qquad f = \frac{mN_0}{\rho_0}$$
$$h = \frac{mN_0 C_{\mathrm{pt}}}{\rho C_1}, \qquad h' = \frac{mN_0 C_{\mathrm{pt}}}{\rho C_1'}$$

Here R_1, S_1, P_r, P_r', M_1, M_1', M_3, M_5, x_1, h and h_1' denote, respectively the modified Rayleigh number, salinity Rayleigh number, Prandtl number, Prandtl number analogous to solute, ratio of magnetic to gravitational forces [buoyancy magnetization], effect on magnetization due to salinity, non-buoyancy magnetization, ratio of the salinity effect on magnetic field to pyromagnetic coefficient, dimensionless wave number, dust particle parameter and dust particles parameter analogous to solute. The non-buoyancy magnetization parameter M_3 measures the departure of linearity in the magnetic equation of state and values from one ($M_0 = \chi H_0$) to higher values are possible for the usual equations of state.

4. Exact solution for free boundaries

Here we consider the case where both boundaries are free as well as perfect conductors of heat. The case of two free boundaries is of little physical interest, but it is mathematically important because one can derive an exact solution, whose properties guide our analysis. Here we consider the case of an infinite magnetic susceptibility χ and we neglect the deformability of the horizontal surfaces. Thus the exact solution of the system (26)–(30) subject to the boundary conditions

$$W^* = D^2W^* = T^* = C^* = D\Phi_1^* = D\Phi_2^* = 0 \quad \text{at } z = \pm\frac{1}{2} \tag{31}$$

is written in the form

$$W^* = A_1 e^{\sigma t^*}\cos\pi z^*, \qquad T^* = B_1 e^{\sigma t^*}\cos\pi z^*$$
$$C^* = F_1 e^{\sigma t^*}\cos\pi z^*, \qquad D\Phi_1^* = C_1 e^{\sigma t^*}\cos\pi z^*$$
$$\Phi_1^* = \frac{C_1}{\pi} e^{\sigma t^*}\sin\pi z^*, \qquad D\Phi_2^* = E_1 e^{\sigma t^*}\cos\pi z^*$$
$$\Phi_2^* = \left(\frac{E_1}{\pi}\right) e^{\sigma t^*}\sin\pi z^* \tag{32}$$

where A_1, B_1, C_1, E_1, F_1 are constants and σ is the growth rate which is, in general, a complex constant.

Substituting Eqs. (32) in Eqs. (26)–(30) and dropping asterisks for convenience, we get following equations

$$\begin{aligned}&\big[\big\{\big(\sigma + (\pi^2+a^2)\big)(1+\tau\sigma) + f\sigma\big\}(\pi^2+a^2)\big]A_1\\ &\quad - \big[aR^{1/2}(1+\tau\sigma)(1+M_1-M_4)\big]B_1\\ &\quad + \big[aR^{1/2}(M_1-M_4)(1+\tau\sigma)\big]C_1\\ &\quad + \big[aS^{1/2}(1+\tau\sigma)(1-M_1'+M_4')\big]F_1\\ &\quad + \big[aS^{1/2}(M_1'-M_4')(1+\tau\sigma)\big]E_1 = 0\end{aligned} \tag{33}$$

$$\begin{aligned}&\big[aR^{1/2}\big\{h + (1-M_2)(1+\tau\sigma)\big\}\big]A_1\\ &\quad - \big[\big\{(\pi^2+a^2) + P_r(1+h)\sigma\big\}(1+\tau\sigma)\big]B_1\\ &\quad + \big[P_r M_2\sigma(1+\tau\sigma)\big]C_1 = 0\end{aligned} \tag{34}$$

$$\begin{aligned}&\big[aS^{1/2}\big\{h' + (1-M_2')(1+\tau\sigma)\big\}\big]A_1\\ &\quad - \big[\big\{(\pi^2+a^2) + P_s(1+h')\sigma\big\}(1+\tau\sigma)\big]F_1\\ &\quad + \big[P_s M_2'\sigma(1+\tau\sigma)\big]E_1 = 0\end{aligned} \tag{35}$$

$$-\pi^2 B_1 + (\pi^2 + a^2 M_3)C_1 = 0 \tag{36}$$

$$-\pi^2 F_1 + (\pi^2 + a^2 M_3)E_1 = 0 \tag{37}$$

For existence of non-trivial solutions of the above equations, the determinant of the coefficients of A_1, B_1, C_1, E_1, F_1 in Eqs. (33)–(37) must vanish. This determinant on simplification yields

$$T_4\sigma_1^4 - iT_3\sigma_1^3 - T_2\sigma_1^2 + iT_1\sigma_1 + T_0 = 0 \tag{38}$$

Here

$$T_4 = \tau_1 b L_2 L_3 \tag{39}$$

$$T_3 = b\big[(\tau_1 b + 1 + f)L_2L_3 + \tau_1 b L_0(L_2+L_3)\big] \tag{40}$$

$$\begin{aligned}T_2 = \big[&\tau_1 b^3 L_0^2 + b^2 L_0(\tau_1 b + 1 + f)(L_2+L_3)\\ &+ b^2 L_2 L_3 - \tau_1 x_1 R_1(1-M_2)L_3L_4\\ &+ \tau_1 x_1 S_1(1-M_2')L_2L_5\big]\end{aligned} \tag{41}$$

$$\begin{aligned}T_1 = \big[&b^3 L_0\big\{L_0[\tau_1 b + 1 + f] + (L_2+L_3)\big\}\\ &- x_1R_1L_4\big[\tau_1(1-M_2)bL_0 + \big\{h + (1-M_2)\big\}L_3\big]\\ &+ x_1S_1L_5\big[\tau_1(1-M_2')bL_0 + \big\{h' + (1-M_2')\big\}L_2\big]\big]\end{aligned} \tag{42}$$

$$\begin{aligned}T_0 = bL_0\big[&b^3L_0 - x_1R_1\big\{h + (1-M_2)\big\}L_4\\ &+ x_1S_1\big\{h' + (1-M_2')\big\}L_5\big]\end{aligned} \tag{43}$$

where

$$R_1 = R/\pi^4, \qquad S_1 = S/\pi^4, \qquad x_1 = a^2/\pi^2$$
$$i\sigma_1 = \sigma/\pi^2, \qquad \tau_1 = \tau\pi^2, \qquad b = (1+x_1)$$
$$L_0 = 1 + x_1M_3, \qquad L_2 = P_r\big[\big\{(1-M_2) + x_1M_3\big\} + L_0h\big]$$
$$L_3 = P_s\big[\big\{(1-M_2') + x_1M_3\big\} + L_0h'\big]$$
$$L_4 = \big\{1 + x_1M_3(1+M_1-M_4)\big\} \quad \text{and}$$
$$L_5 = \big\{1 + x_1M_3(1-M_1'+M_4')\big\}$$

5. The case of stationary convection

When the instability sets in as stationary convection in the case $M_2 \cong 0$ and $M_2' \cong 0$, the marginal state will be characterized by $\sigma_1 = 0$ (Chandrasekhar [13], Finlayson [14]), then the Rayleigh number is given by

$$R_1 = \frac{(1+x_1)^3(1+x_1M_3)}{x_1h_1\{(1+x_1M_3)+x_1M_3M_1(1-M_5)\}} + \frac{S_1h_1'\{(1+x_1M_3)+x_1M_3M_1'(1/M_5-1)\}}{h_1\{(1+x_1M_3)+x_1M_3M_1(1-M_5)\}} \tag{44}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless parameters x_1, M_3, M_5, h_1, h_1' and S_1. Here we put $h_1 = (1+h)$ and $h_1' = (1+h')$. In the absence of dust particles, the values of h_1 and h_1' is one.

A suggestion from Finlayson [14] has also been taken for a variation of these parametric values. In the present analysis, the range of values pertaining to ferric oxide, kerosene and other organic carriers are chosen. With the same ferric oxide, the different carriers like alcohol, hydrocarbon, ester, halocarbon, silicon could be chosen. Depending on this the parametric values of ferrofluid are found to vary within these limits. The value of pyromagnetic coefficient K_2 reported by Rosensweig and Kaiser [51] is $K_2 = 0.03$ gauss·°C^{-1} ($= 30$ A·M·°C^{-1}) for magnetite in kerosene. The magnetic forces can be increased by increasing K_2, either by using higher concentration or by suspending a solid with a higher value of K_2. In very thin layers (less than 1 mm for the fluids) only the magnetic forces contribute to convection (Finlayson [14]). For such fluids, the typical values of M_2 are 10^{-6} and so is assumed to have a negligible value and hence it is taken to be zero. M_3 is varied from 1 to 25. The salinity Rayleigh number S_1 is varied from 0 to +500. The buoyancy magnetization parameter M_1 is assumed to be 1000. M_1' is allowed to vary from 0.1 to 0.5 taking values less than the non-buoyancy magnetization parameter M_3. M_5 is varied between 0.1 and 0.5.

To investigate the effects of non-buoyancy magnetization, dust particles and stable solute gradient, we examine the behavior of $\mathrm{d}R_1/\mathrm{d}M_3$, $\mathrm{dR}_1/\mathrm{d}h_1$ and $\mathrm{d}R_1/\mathrm{d}S_1$ analytically. Eq. (44) yields

$$\frac{\mathrm{d}R_1}{\mathrm{d}M_3} = -\big\{(1-M_5)\big[(1+x_1)^3M_1 + x_1S_1h_1'\{M_1 - M_1'/M_5\}\big]\big\} \times \big\{h_1\big\{(1+x_1M_3)+x_1M_3M_1(1-M_5)\big\}^2\big\}^{-1} \tag{45}$$

$$\frac{\mathrm{d}R_1}{\mathrm{d}h_1} = -\big\{(1+x_1)^3(1+x_1M_3) + x_1S_1h_1'\big\{(1+x_1M_3)+x_1M_3M_1'(1/M_5-1)\big\}\big\} \times \big\{x_1h_1^2\big\{(1+x_1M_3)+x_1M_3M_1(1-M_5)\big\}\big\}^{-1} \tag{46}$$

$$\frac{\mathrm{d}R_1}{\mathrm{d}S_1} = \frac{h_1'[(1+x_1M_3)+x_1M_3M_1'\{1/M_5-1\}]}{h_1\{(1+x_1M_3)+x_1M_3M_1(1-M_5)\}} \tag{47}$$

The destabilizing effect of the non-buoyancy magnetization and dust particles is evident from the fact that $\mathrm{d}R_1/\mathrm{d}M_3$ and $\mathrm{d}R_1/\mathrm{d}h_1$ are always negative, i.e. the Rayleigh number decreases with an increase in non-buoyancy magnetization and dust particles parameters. Eq. (47) yields that $\mathrm{d}R_1/\mathrm{d}S_1$ is always positive, thus indicating the stabilizing effect of solute gradient. For sufficiently large values of M_1 (Finlayson [14]), we obtain the results for the magnetic mechanism

$$R_m = R_1M_1 = \frac{(1+x_1)^3(1+x_1M_3)}{x_1^2h_1M_3(1-M_5)} + \frac{S_1h_1'\{1+x_1M_3+x_1M_1'M_3(1/M_5-1)\}}{x_1h_1M_3(1-M_5)} \tag{48}$$

where R_m is the magnetic thermal Rayleigh number.

As a function of x_1, R_m given by Eq. (48) attains its minimum when

$$2M_3x_1^4 + (1+3M_3)x_1^3 - (M_3 + S_1h_1' + 3)x_1 - 2 = 0 \tag{49}$$

The values of critical wave number for the onset of instability are determined numerically using Newton-Raphson method by the condition $\mathrm{d}R_m/\mathrm{d}x_1 = 0$. With x_1 determined as a solution of Eq. (49), Eq. (48) will give the required critical magnetic thermal Rayleigh number N_c. The critical magnetic thermal Rayleigh number (N_c), depends on the non-buoyancy magnetization parameter M_3, ratio of the salinity effect on magnetic field to pyromagnetic coefficient M_5, dust particles parameter h_1, dust particles parameter analogous to solute h_1' and stable solute gradient S_1. Values of N_c determined for various values of M_3, h_1, h_1' and S_1 are given in Table 1 and the results are further illustrated in Figs. 2, 3 and 4.

Figs. 2(a) and 2(b) represent the plots of critical wave number x_c and critical magnetic thermal Rayleigh number N_c versus non-buoyancy magnetization parameter M_3 for various values of h_1. Fig. 2(a) indicates the destabilizing nature of cell size x_c as M_3 increases. Fig. 2(b) illustrates that as non-buoyancy magnetization parameter increases, the critical magnetic Rayleigh number N_c decreases. This is because variation in magnetization releases extra energy which adds up to thermal energy to destabilize the system. Therefore the system will always be in convective mode even for the smallest thermal and magnetic gradients. Therefore, lower values of N_c are needed for onset of convection with an increase in M_3, hence justifying the destabilizing effects of non-buoyancy magnetization. Also, it is observed from Fig. 2(b) that as value of M_3 is less than 6, the critical magnetic Rayleigh number N_c shows a drastic decrease leading to a value of 220 and has less influence in the value of N_c as magnetization M_3 is increased further.

Figs. 3(a) and 3(b) give the variation of critical wave number x_c and critical magnetic thermal Rayleigh number N_c versus dust particles parameters (h_1, h_1') for different values of S_1. These figures show the stabilizing nature of cell

Table 1
Critical magnetic thermal Rayleigh numbers and wave numbers of the unstable modes at marginal stability for the onset of stationary convection for various values of solute gradient, non-buoyancy magnetization and dust particles parameters

S_1	M_3	$M_5=0.1$, $M_1'=0.1$							
		$h_1=1, h_1'=1$		$h_1=3, h_1'=3$		$h_1=5, h_1'=5$		$h_1=7, h_1'=7$	
		x_c	N_c	x_c	N_c	x_c	N_c	x_c	N_c
0	1	1.00	17.78	1.00	5.93	1.00	3.56	1.00	2.54
	5	0.69	10.03	0.69	3.34	0.69	2.01	0.69	1.43
	10	0.61	8.85	0.61	2.95	0.61	1.77	0.61	1.26
	15	0.58	8.42	0.58	2.81	0.58	1.68	0.58	1.20
	20	0.56	8.20	0.56	2.73	0.56	1.64	0.56	1.17
100	1	3.18	279.60	4.75	252.45	5.72	244.39	6.45	240.09
	5	1.79	238.51	2.68	226.80	3.24	223.52	3.67	221.81
	10	1.39	230.80	2.08	221.91	2.51	219.52	2.85	218.30
	15	1.21	227.69	1.79	219.91	2.16	217.89	2.45	216.87
	20	1.09	225.94	1.61	218.78	1.94	216.96	2.20	216.05
200	1	4.10	521.14	6.11	483.95	7.33	472.45	8.27	466.23
	5	2.31	460.42	3.47	445.11	4.18	440.60	4.72	438.20
	10	1.79	448.87	2.69	437.66	3.25	434.47	3.68	432.80
	15	1.54	444.18	2.32	434.62	2.80	431.97	3.17	430.60
	20	1.39	441.52	2.08	432.89	2.52	430.55	2.85	429.35
300	1	4.75	757.34	7.06	711.91	8.47	697.60	9.55	689.80
	5	2.68	680.41	4.02	662.15	4.84	656.63	5.46	653.67
	10	2.08	665.72	3.13	652.58	3.77	648.74	4.26	646.71
	15	1.79	659.73	2.69	648.68	3.25	645.53	3.68	643.87
	20	1.61	656.34	2.42	646.47	2.93	643.70	3.31	642.27
400	1	5.27	990.61	7.83	937.93	9.38	921.15	10.56	911.95
	5	2.99	899.36	4.46	878.50	5.37	872.09	6.06	868.63
	10	2.31	881.88	3.48	867.06	4.19	862.64	4.73	860.29
	15	1.99	874.76	3.00	862.40	3.62	858.79	4.09	856.89
	20	1.79	870.72	2.69	859.75	3.25	856.61	3.68	854.96

size x_c, whereas destabilizing nature of N_c as both h_1, h_1' increase. In the absence of solute gradient, constant value of x_c is observed as h_1 increases, whereas in the presence of solute gradient, the value of x_c increases as h_1 increases, showing the stabilizing nature of cell size x_c. Therefore, due to the presence of stable solute gradient, there is a competition of stabilizing role of stable solute gradient and destabilizing role of dust particles. Here the critical stability parameter N_c is reduced in the presence of dust particles because the heat capacity of clean fluid is supplemented by that of the dust particles. This new type of phenomenon is observed here.

Figs. 4(a) and 4(b) give the variation of critical wave number x_c versus solute gradient S_1 and consequent critical magnetic Rayleigh number N_c versus solute gradient S_1 for different values of h_1 and h_1'. These figures show the destabilizing nature of cell size x_c and corresponding N_c as S_1 increases. Thus, these graphs exhibit a destabilizing trend.

6. The case of oscillatory modes

Here we examine the possibility of oscillatory modes, if any, on stability problem due to the presence of dust particles, stable solute parameter and magnetization parameter. Equating the imaginary parts of Eq. (38), we obtain

$$\begin{aligned}\sigma_1\big[b^3L_0\{L_0\{\tau_1 b+1+f\}+(L_2+L_3)\}\\ -x_1R_1L_4\big[\tau_1(1-M_2)bL_0+\{h+(1-M_2)\}L_3\big]\\ +x_1S_1L_5\big[\tau_1(1-M_2')bL_0+\{h'+(1-M_2')\}L_2\big]\\ -\sigma_1^2b\big[L_2L_3(\tau_1 b+1+f)+\tau_1 bL_0(L_2+L_3)\big]\big]=0\end{aligned} \tag{50}$$

It is evident from Eq. (50) that σ_1 may be either zero or non-zero, meaning that the modes may be either non-oscillatory or oscillatory. In the absence of dust particles and stable solute gradient, we obtain the above result as

$$\sigma_1\big[P_r\{(1-M_2)+x_1M_3\}+(1+x_1M_3)\big]=0 \tag{51}$$

Here the quantity inside the brackets is positive definite because the typical values of M_2 are $+10^{-6}$ (Finlayson [14]). Hence

$$\sigma_1=0 \tag{52}$$

which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied, in the absence of dust particles and stable solute gradient. Thus from Eq. (50), we conclude that the oscillatory modes are introduced due to the presence of the dust particles and stable solute gradient, which were non-existent in their absence.

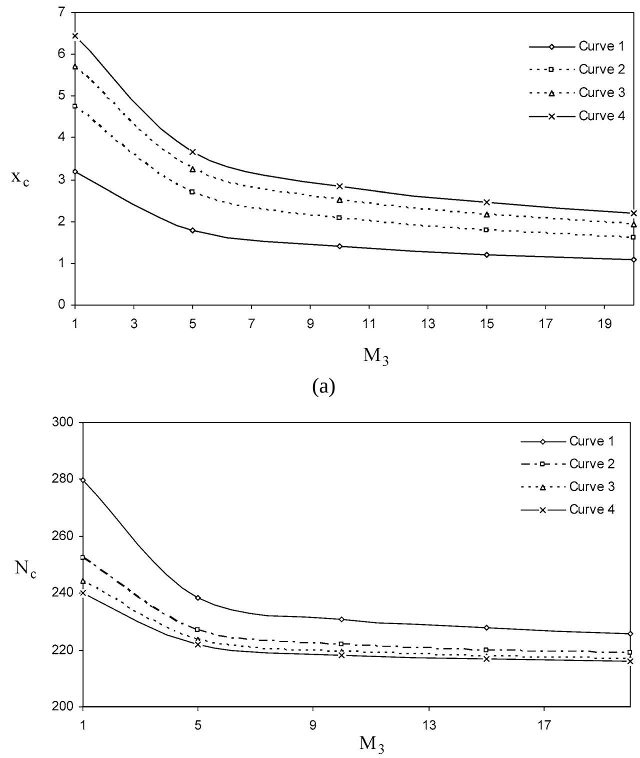

Fig. 2. (a) Variation of x_{c} versus M_3. (b) Marginal instability curve for variation of N_{c} versus M_3 for $S_1 = 100$, $M_5 = 0.1$, $M_1' = 0.1$; $h_1 = 1$ for curve 1, $h_1 = 3$ for curve 2, $h_1 = 5$ for curve 3 and $h_1 = 7$ for curve 4.

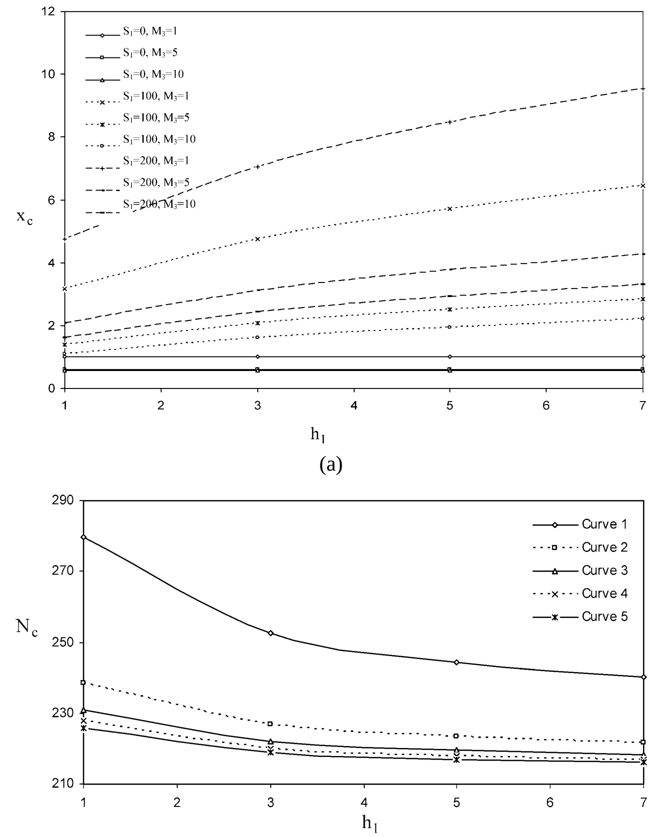

Fig. 3. (a) Variation of x_{c} versus h_1. (b) Marginal instability curve for variation of N_{c} versus h_1 for $M_3 = 1$, $M_5 = 0.1$, $M_1' = 0.1$; $S_1 = 0$ for curve 1, $S_1 = 100$ for curve 2, $S_1 = 200$ for curve 3, $S_1 = 300$ for curve 4 and $S_1 = 400$ for curve 5.

7. The case of overstability

The present section is devoted to find the possibility that the observed instability may really be overstability. Since we wish to determine the Rayleigh number for the onset of instability through state of pure oscillations, is sufficient to find conditions for which Eq. (38) will admit of solutions with σ_1 real.

Equating real and imaginary parts of Eq. (38) and eliminating R_1 between them, we obtain

$$A_2 c_1^2 + A_1 c_1 + A_0 = 0 \tag{53}$$

where $c_1 = \sigma_1^2$

$$\begin{aligned} A_2 = {} & \tau_1 b L_3^2 \bigl[\bigl\{\tau_1(1-M_2)(L_0+L_2)\bigr\} b \\ & + L_2\bigl\{f(1-M_2)-h\bigr\}\bigr] \end{aligned} \tag{54}$$

$$\begin{aligned} A_1 = {} & \bigl\{\bigl[\tau_1^2 L_0^2(1-M_2)(L_0+L_2)\bigr] b^4 \\ & + \bigl[\tau_1 L_0^2 L_2\bigl\{f(1-M_2)-h\bigr\} + \tau_1 h L_0 L_3^2\bigr] b^3 \\ & + L_3^2\bigl\{L_0(1+f)+L_2\bigr\}\bigl[h+(1-M_2)\bigr] b^2 \\ & + \bigl[L_0 L_5 \tau_1^2 x_1 (1-M_2)(1-M_2') S_1 (L_2-L_3)\bigr] b \\ & + \bigl[\tau_1 x_1 S_1 L_2 L_3 L_5 \bigl\{h(1-M_2') - h'(1-M_2)\bigr\}\bigr]\bigr\} \end{aligned} \tag{55}$$

$$\begin{aligned} A_0 = {} & b L_0 \bigl\{\bigl(L_0^2 h \tau_1\bigr) b^4 \\ & + L_0\bigl[\bigl\{h+(1-M_2)\bigr\}\bigl\{(1+f)L_0+L_2\bigr\}\bigr] b^3 \\ & + \bigl[\tau_1 x_1 L_0 L_5 S_1 \bigl\{h(1-M_2') - h'(1-M_2)\bigr\}\bigr] b \\ & + \bigl[x_1 L_5 S_1 \bigl\{h+(1-M_2)\bigr\}\bigl\{h'+(1-M_2')\bigr\} \\ & \times (L_2-L_3)\bigr]\bigr\} \end{aligned} \tag{56}$$

Since σ_1 is real for overstability, both values of c_1 $(=\sigma_1^2)$ are positive. Eq. (53) is quadratic in c_1 and does not involve any of its roots to be positive if

$$f > \frac{h}{1-M_2}, \quad \frac{h}{1-M_2} > \frac{h'}{1-M_2'} \quad \text{and} \quad L_2 > L_3 \tag{57}$$

i.e. if

$$f > \frac{h'}{1-M_2'} \quad \text{and} \quad L_2 > L_3 \tag{58}$$

i.e. if

$$\begin{aligned} & (\rho_0 C_{\mathrm{V,H}} - \mu_0 K_3 H_0)(1-M_2') > \rho_0 C_{\mathrm{pt}}, \\ & P_r > P_s, \quad P_r > P_s + P_r M_2 \quad \text{and} \quad h P_r > h' P_s \end{aligned} \tag{59}$$

which implies that

$$\begin{aligned} & (\rho_0 C_{\mathrm{V,H}} - \mu_0 K_3 H_0)(1-M_2') > \rho_0 C_{\mathrm{pt}}, \\ & P_r > P_s + P_r M_2, \qquad h P_r > h' P_s \end{aligned} \tag{60}$$

and the other inequality $P_r > P_s$ being automatically satisfied in view of (60).

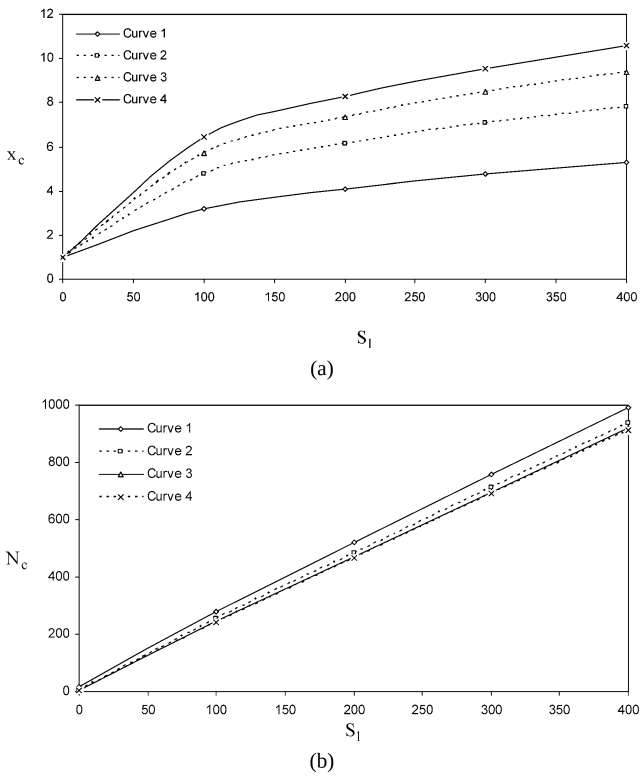

Fig. 4. (a) Variation of x_c versus S_1. (b) Marginal instability curve for variation of N_c versus S_1 for $M_3 = 1$, $M_5 = 0.1$, $M_1' = 0.1$; $h_1 = 1$ for curve 1, $h_1 = 3$ for curve 2, $h_1 = 5$ for curve 3 and $h_1 = 7$ for curve 4.

Thus, for $(\rho_0 C_{V,H} - \mu_0 K_3 H_0)(1 - M_2') > \rho_0 C_{pt}$, $P_r > P_s + P_r M_2$, $h P_r > h' P_s$, overstability cannot occur and the principle of the exchange of stabilities is valid. Hence the above conditions are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability. In the absence of magnetic parameters (in non-magnetic fluid) the above conditions, as expected, reduce to $C_v > C_{pt}$, i.e. the specific heat of fluid at constant volume is greater than the specific heat of dust particles and $K_1 < K_1'$, i.e. the thermal conductivity is less than the solute conductivity, which is in good agreement with the results obtained earlier [43–46]. In the presence of magnetic parameters but in the absence of dust particles the above conditions, as expected, reduce to

$$P_r > \frac{P_s}{1 - M_2} \quad \text{i.e.}$$

$$K_1' > K_1 \left[\frac{\rho C_1'}{\rho C_1 \{1 - \mu_0 T_0 K_2^2 / ((1 + \chi)\rho C_1)\}} \right],$$

which is also in good agreement with the results obtained earlier by Sunil et al. [38].

8. Discussion of results and conclusions

In this paper, we studied the effects of dust particles on a ferrofluid heated and soluted from below in the presence of uniform vertical magnetic field. We have investigated the effects of non-buoyancy magnetization, stable solute gradient and dust particles on the onset of convection. The principal conclusions from the analysis of this paper are as under:

(i) For the case of stationary convection, the non-buoyancy magnetization, dust particles always have a destabilizing effect, whereas stable solute gradient delays the onset of convection as is evident from Eq. (47).

(ii) The critical wave numbers and critical magnetic thermal Rayleigh numbers for the onset of instability are also determined numerically for sufficiently large values of buoyancy magnetic parameter M_1 and the results are depicted graphically. The effects of governing parameters on the stability of the system are discussed below.
- The destabilizing nature of cell size (x_c) and corresponding N_c as non-buoyancy magnetization M_3 increases can be observed from Figs. 2(a) and 2(b) and also from Table 1 for other different values.
- Table 1 and Figs. 3(a) and 3(b) lead to the conclusion that cell size always has a stabilizing nature, whereas dust particles have always a destabilizing nature. Therefore, lower the value of N_c earlier will be onset of convection with an increase in h_1. The destabilizing effect of dust particles on non-magnetic fluid is accounted by many authors [42–49] and is found to be valid for a ferrofluid also.
- We have also looked into the effect of stable solute gradient S_1. Figs. 4(a) and 4(b) demonstrate the effect of S_1 on x_c and N_c. Fig. 4(a) shows the destabilizing nature of cell size x_c. It is also observed from Fig. 4(b) that a stable solute gradient delays the onset of convection. This is in contrast to the case of "soluted from above" where a unstable solute gradient have the destabilizing effect on the system (Vaidyanathan et al. [28,29]).
- We also observed that the critical stability parameter, N_c is reduced in the presence of dust particles because the heat capacity of clean fluid is supplemented by that of the dust particles.

(iii) The principle of exchange of stabilities is found to hold true for the ferrofluid heated from below in the absence of dust particles and stable solute gradient. The oscillatory modes are introduced due to the presence of the dust particles and stable solute gradient, which were non-existent in their absence.

(iv) The conditions $(\rho_0 C_{V,H} - \mu_0 K_3 H_0)(1 - M_2) > \rho_0 C_{pt}$, $P_r > P_s + P_r M_2$, $h P_r > h' P_s$ are sufficient for the non-existence of overstability. In the absence of magnetic parameters the above conditions, as expected, reduces to $C_v > C_{pt}$, i.e. the specific heat of fluid at constant volume is greater than the specific heat of dust particles and $K_1 < K_1'$, i.e. the thermal conductivity is less than the solute conductivity, which is in good agreement with the previous published work.

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